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LETTER TO THE EDITOR

The stochastic mechanics of fields in a general relativistic context: problems and perspectives

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Abstract. The problem of a formulation of Nelson's stochastic mechanics of scalar fields in the context of general relativity is considered. The simple example of the scalar field in the Wightman vacuum state on the Rindler wedge is examined, the stochastic counterpart of the Fulling ambiguity of canonical quantisation is formulated, and the role of the stochastic mechanics of thermal mixtures, as formulated by Guerra and Loffredo, is analysed, in the spirit of Davies and Unruh, in the solution of the above ambiguity. An overall picture emerges which, both in the explicit example considered here and in its straightforward generalisations to static submanifolds of more general spacetimes, confirms Smolin's point of view that stochastic quantisation is a very natural conceptual frame in which to study the general non-covariance of the distinction between quantum and thermal fluctuations.

The difficulties presented and the perspectives offered by a solution of the problem (posed as problem 13 in the conclusions of [1]) 'to formulate stochastic mechanics in the context of general relativity' are best exemplified by the case of the Schwarzschild metric:

$$ds^2 = -(1-2M/r)\,dt^2 + (1-2M/r)^{-1}\,dr^2 + r^2(d\theta^2 + \sin^2\theta\,d\varphi^2).$$

Examination of the curvature invariant [2]

$$R_{\alpha\beta\gamma\delta}R^{\alpha\beta\gamma\delta} = 48M^2/r^6 \tag{1}$$

shows that $r=0$ is an actual singularity, while $r=2M$ is a coordinate singularity, which can in fact be eliminated by the adoption, for instance, of Kruskal coordinates [3]:

$$x^1 = \varphi(r)\begin{cases}\sinh t/4M & r<2M\\ \cosh t/4M & r>2M\end{cases}$$

$$x^0 = \varphi(r)\begin{cases}\cosh t/4M & r<2M\\ \sinh t/4M & r>2M\end{cases}$$

$$\varphi(r) = |(r/2M)-1|\exp(r/4M).$$

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In terms of these coordinates, the exterior Schwarzschild region ($r > 2M$, $t \in \mathbb{R}$) appears as a static *proper* submanifold, characterised by $x^1 > |x^0|$, of Kruskal spacetime:

$$(x^0)^2 - (x^1)^2 < 1$$
$$ds^2 = (32M^3/r)\exp(-r/2M)[-(dx^0)^2 + (dx^1)^2] + r^2(d\theta^2 + \sin^2\theta\, d\varphi^2)$$
$$(r/2M - 1)\exp(r/2M) = (x^1)^2 - (x^0)^2.$$

At $r = 2M$ the vectors $\partial/\partial r$, $\partial/\partial t$ exchange their roles as spacelike and timelike vectors (so that 'the unseen power of the world which drags everyone forward willy-nilly from age twenty to forty and from forty to eighty also drags the rocket in from time coordinate $r = 2M$ to the later value of the time coordinate $r = 0$' [2] to be crushed then by the infinite tidal stress described by equation (1)). Intimately related to this exchange is the fact that the exterior Schwarzschild wedge $W_S = \{x^1 > |x^0|\}$ in Kruskal spacetime is, with no region $\mathscr{A}$ in its complement, in the *mutual* causal relationship (send *and* receive light signals) necessary to give operational meaning, with respect to W_S, to the notion of observables localised in $\mathscr{A}$. Furthermore, one cannot rule out in an operationally meaningful way situations in which signals (energy and entropy) flow into W_S through the past horizon $x^0 = -x^1$ and out of W_S through the future horizon $x^0 = x^1$, so that without any operational meaning there appears in particular the preparation of quantum states of fields in W_S describing complete decoupling of W_S from its complement.

Hawking [4] has recognised that, in the quite generic situation of spacetime singularities under the 'censorship' of event horizons, physically realisable states of the quantum field algebra on, say, W_S are to be considered as those for which the degrees of freedom of the field 'behind the horizon' appear as a 'thermal reservoir'. This recognition has introduced in physics a second instance (the first being, of course, quantum mechanics) in which probabilistic notions impose themselves for much more fundamental reasons than accidentally partial knowledge of the system. As observed by Smolin [5, 6], Nelson's stochastic mechanics [1] (considered here in its generalisation to fields given by Guerra and Ruggiero [7]) presents itself, precisely because it treats quantum fluctuations themselves in ordinary classical probabilistic language, as a natural framework in which to attempt a synthesis of the above two instances.

The explicit example we consider here is the quantum theory on Minkowski spacetime of a scalar field $\hat{\varphi}$, restricted to the Rindler wedge [8] W_R, for which, relative to the vacuum state Ω, Hawking's 'thermal ansatz' at the Davies-Unruh [9, 10] temperature has been shown by Sewell [11] to be an intrinsic structural property of a theory satisfying Wightman axioms. A similar scenario has been analysed from the axiomatic point of view [11] for a class of spacetimes including the Schwarzschild wedge in Kruskal spacetime and examined by scaling arguments for states of quantum fields on a gravitational background satisfying the Haag-Narnhofer-Stein [12, 13] conditions of local definiteness and local stability.

In considering such structural properties in the language of stochastic field theory (the analysis of whose detailed mathematical structure is just now beginning [14]) we note the lack of stochastic analogues of such deep results as the Reeh-Schlieder [15], Tomita-Takesaki [16] and Bisognano-Wichman [17] theorems which are included in Sewell's arguments (the attempt to foreshadow the form of such stochastic analogues is, in fact, the main motivation of this research). This forces us to restrict ourselves to the simplest explicit example available; the lack of mathematical depth and generality imposes, however, in stochastic field theory, a sharper operational understanding of the problems and the results of an analysis of stochastic fields in non-inertial frames.

It is in particular suggestive to read the result of the discussion that follows (equation (6)) as a simple 'stochastic form of the law of inertia' [5]. Inertial frames are those in which total fluctuations are a minimum (the Guerra-Ruggiero [7] coincidence of the ground-state process with the Euclidean process being a signal of this minimality); in general non-inertial frames 'thermal' fluctuations appear superimposed onto the minimal quantum level.

We start by reviewing some elementary kinematical facts [2].

Let $A(\tau)$ be a world line in (for notational simplicity, two-dimensional) Minkowski spacetime $\mathbb{M}^2$, parametrised by proper time τ. At each point on this world line a normalised timelike vector

$$u(\tau) = \mathrm{d}A(\tau)/\mathrm{d}\tau$$

and a normalised spacelike vector

$$v(\tau) = (1/g)\,\mathrm{d}u(\tau)/\mathrm{d}\tau$$

(where $g(\tau) = (\mathrm{d}u/\mathrm{d}\tau)(\mathrm{d}u/\mathrm{d}\tau)$, so that $v \cdot v = 1$) are available and $u \cdot v = 0$.

Each event P in $\mathbb{M}^2$ for which there exists a unique τ_P such that

$$(P - A(\tau_P))u(\tau_P) = 0$$

is uniquely determined by its 'coordinates in the moving frame carried by the observer $A(\tau)$':

$$(t, r) \equiv (\tau_P, (P - A(\tau_P)) \cdot v(\tau_P)).$$

Suppose, in particular, $g(\tau) = \text{constant} = g$. The above coordinates of P are then, with the coordinates (x^0, x^1) assigned to P (by essentially the same operational procedure) by an inertial observer $I(\tau)$ such that

$$I = A \qquad \mathrm{d}I/\mathrm{d}\tau = \mathrm{d}A/\mathrm{d}\tau \qquad \text{for } \tau = 0$$

in the relation

$$x^0 = (1/g + r)\sinh gt \qquad x^1 + 1/g = (1/g + r)\cosh gt.$$

The above notational scheme has been momentarily adopted here both to stress the Rindler-Scharzschild and Minkowski-Kruskal analogies and to make the coincidence of the inertial and non-inertial coordinates evident in the limit $g \to 0$.

A space translation $x^1 \to x^1 + 1/g$ and the renaming

$$(1/g + r) \equiv \xi \qquad gt \equiv \tau$$

sets the above coordinate relations in the conventional Rindler form:

$$x^0 = \xi \sinh \tau \qquad x^1 = \xi \cosh \tau.$$

Notice that

$$\mathrm{d}s^2 = -(\mathrm{d}x^0)^2 + (\mathrm{d}x^1)^2 = -\xi^2\,\mathrm{d}\tau^2 + \mathrm{d}\xi^2$$

$$\Box = -\frac{\partial^2}{\partial x^{0^2}} + \frac{\partial^2}{\partial x^{1^2}} = -\frac{1}{\xi^2}\frac{\partial^2}{\partial \tau^2} + \frac{\partial^2}{\partial \xi^2} + \frac{1}{\xi}\frac{\partial}{\partial \xi}.$$

The fact that $\partial(x^0, x^1)/\partial(\tau, \xi) = \xi$ so that the Rindler chart

$$\mathcal{R}\colon P \in W_{\mathrm{R}} = \{x^1 > |x^0|\} \to \mathcal{R}(P) = (\tau, \xi)$$

becomes singular on ∂W_R, reflects of course the fact that the above procedure to define non-inertial coordinates becomes operationally meaningless. This is due to the lack of mutual causal relationships for events which are, with respect to the hyperbolic world line $A(\tau)$, on opposite sides of its asymptotes which therefore act as event horizons.

The above considerations have been reviewed in all their elementary details in order to show that the spacetime manifold W_R has, considered in its own right, precisely the same operational meaning and limitations as the exterior Schwarzschild region W_S, again considered by itself.

We now set up the following problem. Find a Gaussian stationary random field $\varphi_{\mathscr{R}}(\tau, \xi)$ on W_R such that:

(I) the Markov property is satisfied by its dependence on τ,

(II) the Klein–Gordon equation

$$\left(\frac{1}{\xi^2}D_\tau^2 - \frac{\partial^2}{\partial \xi^2} - \frac{1}{\xi}\frac{\partial}{\partial \xi} + m^2\right)\varphi_{\mathscr{R}}(\tau, \xi) = 0$$

is satisfied, the second derivative with respect to τ being taken in the symmetrised smoothed sense of Nelson [18]

$$D_\tau^2 = \tfrac{1}{2}(D_\tau^+ D_\tau^- + D_\tau^- D_\tau^+)$$

where $D_\tau^\pm$ are mean forward $(+)$ and backward $(-)$ derivatives with respect to τ, and

(III) the scale of the randomness of the time evolution in τ is set by the stochastic analogue of the equal-time commutation relations [19]

$$E(\varphi_{\mathscr{R}}(\tau, \xi)(D_\tau^- - D_\tau^+)\varphi_{\mathscr{R}}(\tau, \xi')) = \xi\delta(\xi - \xi').$$

The above conditions are as literal a translation on W_R of stochastic field quantisation on the whole of $\mathbb{M}^2$ (in the approach of [7, 20]) as is Fulling's discussion [21] of canonical field quantisation. We will see, of course, that the same problems appear in the stochastic language.

The problem posed above has a unique solution of the form

$$\varphi_{\mathscr{R}}(\tau, \xi) = \int_0^\infty \mathrm{d}\omega\, q_\omega^{\mathscr{R}}(\tau)\psi_\omega(\xi)$$

where the ψ_ω are Fulling normal modes (equation (22) of [21]):

$$\psi_\omega(\xi) = \pi^{-1}(2\omega \sinh \pi\omega)^{1/2} K_{\mathrm{i}\omega}(m\xi). \tag{2}$$

Here the constant in front of the modified Bessel function [22] $K_{\mathrm{i}\omega}$ has been chosen in such a way that orthonormality and completeness are, respectively,

$$\int_0^\infty \frac{\mathrm{d}\xi}{\xi}\psi_\omega(\xi)\psi_{\omega'}(\xi) = \delta(\omega - \omega')$$

$$\int_0^\infty \mathrm{d}\omega\, \psi_\omega(\xi)\psi_\omega(\xi') = \xi\delta(\xi - \xi').$$

Another set of solutions, linearly independent from those above, of the modified Bessel equations which determine the ψ_ω has been discarded on the basis of exponential growth as $\xi \to \infty$.

The $q_\omega^{\mathscr{R}}$, being independent Gaussian Markovian stationary processes, must have covariance of the form [23]:

$$E(q_\omega^{\mathscr{R}}(\tau)q_\omega^{\mathscr{R}}(\tau')) = a(\omega)\exp(-b(\omega)|\tau-\tau'|)\delta(\omega-\omega').$$

$b(\omega)$ is fixed to be equal to ω by the dynamical condition II which imposes

$$D_\tau^2 q_\omega^{\mathscr{R}}(\tau) + \omega^2 q_\omega^{\mathscr{R}}(\tau) = 0.$$

$a(\omega)$ is fixed to be equal to $1/2\omega$ by condition III which imposes

$$E(q_\omega^{\mathscr{R}}(\tau)(D_\tau^- - D_\tau^+)q_\omega^{\mathscr{R}}(\tau)) = 1.$$

Summarising, $\varphi_{\mathscr{R}}(\tau, \xi)$ is the Gaussian field on W_R with covariance

$$E(\varphi_{\mathscr{R}}(\tau,\xi)\varphi_{\mathscr{R}}(\tau',\xi')) = \int_0^\infty \mathrm{d}\omega\, \exp(-\omega|\tau-\tau'|)\psi_\omega(\xi)\psi_\omega(\xi')/2\omega.$$

We compare now $\varphi_{\mathscr{R}}$ with the result φ_Σ of the discussion of [7, 20] on the whole of $\mathbb{M}^2$, referred to an inertial chart

$$\Sigma\colon P\in\mathbb{M}^2 \to \Sigma(P) = (x^0, x^1).$$

The explicit expression of the Guerra-Ruggiero covariance

$$\begin{aligned} E(\varphi_\Sigma(x^0, x^1)\varphi_\Sigma(x^{0\prime}, x^{1\prime})) &= (2\pi)^{-2}\int \mathrm{d}^2K \frac{\exp\{\mathrm{i}[K^0(x^0-x^{0\prime}) + K^1(x^1-x^{1\prime})]\}}{(K^0)^2+(K^1)^2+m^2} \\ &= (2\pi)^{-1}K_0 m[(x^0-x^{0\prime})^2 + (x^1-x^{1\prime})^2]^{1/2} \end{aligned}$$

shows that the two stochastic processes $\varphi_{\mathscr{R}}$ and φ_Σ are different, although apparently constructed starting from the same local rquirements (I, II, III, formulated for each process in the appropriate coordinates†) are different. In particular, even for events P and Q simultaneous for both the $\mathscr{R}$ and Σ observer, it is

$$E(\varphi_{\mathscr{R}}(P)\varphi_{\mathscr{R}}(Q)) \neq E(\varphi_\Sigma(P)\varphi_\Sigma(Q)).$$

As observed in [24], indeed, $\varphi_{\mathscr{R}}$ is the stochastic process associated, in the sense of Féynes [25] and Nelson [18], to the Fulling state F (the one annihilated, in the canonical formalism, by the annihilation operators of the modes of equation (2)):

$$E(\varphi_{\mathscr{R}}(A)\varphi_{\mathscr{R}}(B)) = \langle F, \hat\varphi(A)\hat\varphi(B)F\rangle \qquad A, B \text{ are } \mathscr{R}\text{-simultaneous}$$

while φ_Σ is the stochastic process associated with the Wightman vacuum state Ω:

$$E(\varphi_\Sigma(c)\varphi_\Sigma(D)) = \langle\Omega, \hat\varphi(c)\hat\varphi(D)\Omega\rangle \qquad C, D \text{ are } \Sigma\text{-simultaneous.}$$

The problem therefore emerges to find a natural way to define a Gaussian stationary (with respect to the Rindler time τ) random field φ on the Rindler wedge $W_\mathrm{R} = \{\tau\in\mathbb{R}, \xi>0\}$ such that, for any two $\mathscr{R}$-simultaneous events P and Q in W_R,

$$E(\varphi(P)\varphi(Q)) = \langle\Omega, \hat\varphi(P)\hat\varphi(Q)\Omega\rangle. \tag{3}$$

The difficulty is, of course, in the uniqueness of the solution $\varphi_{\mathscr{R}}$ of conditions I, II and III, which turned out not to satisfy equation (3).

† The role of the analogue of condition III in establishing the covariance of the normal mode amplitude is taken in [7, 20] to the same effect by the explicit requirement that each normal mode amplitude should perform its 'ground state process'. We have preferred the local condition III, which can, apparently, be formulated without any reference to a globally defined 'vacuum state'.

Notice that condition III is a condition on the diffusion coefficient of the stochastic evolution. This is a state-independent requirement which it is logically inconsistent to drop to make contact with a particular state as in equation (3) (even accepting this, the required changes would turn out to be different for different normal modes).

We explore here the possibility of relaxing condition II in the rather minimal way of requiring that the field φ be constructed as a mixture of solutions of II, with the required reinterpretation of condition I as in [26]. We do this, of course, with the hindsight of the Davies-Unruh-Sewell analysis and of the simple but penetrating observation by Jaekel and Pignon [27] that the mixture of two solutions of the Newton-Nelson equation need not be a solution of the same equation. Possible modifications of the dynamics, in the direction of formulating a stochastic form of the classical KMS boundary conditions have been analysed by Vilela Mendes [28].

We observe, first of all, that the field $\varphi_{\mathscr{R}}$ constructed above describes complete decoupling of W_R from the rest of $\mathbb{M}^2$. Indeed, for $Q \in W_R$

$$\lim_{P \to \partial W_R} E(\varphi_{\mathscr{R}}(P)\varphi_{\mathscr{R}}(Q)) = 0.$$

This can be easily seen from the fact that as $P \equiv (\tau, \xi)$ tends to the horizon ∂W_R, $\xi \to 0$, $\tau \to \pm\infty$ ($\xi\, e^{\tau}$ or $\xi\, e^{-\tau}$ tending to a finite limit) and from the fact that

$$K_{i\omega}(\rho) \underset{\rho \to 0}{\sim} \pi\delta(\omega)$$

(equation (3.27) of [12]).

For comparision, consider the ordinary Wightman function

$$W(P, Q) = \langle \Omega, \hat{\varphi}(P)\hat{\varphi}(Q)\Omega \rangle$$

written, for P and Q in W_R, in terms of Rindler coordinates (see the appendix)

$$W(P, Q) = \int_0^\infty d\omega\, f_\omega(\tau, \tau')\psi_\omega(\xi)\psi_\omega(\xi') \tag{4}$$

where

$$f_\omega(\tau, \tau') = \frac{\exp[-i\omega(\tau - \tau')]}{2\omega} + \frac{\cos \omega(\tau - \tau')}{\omega(\exp \beta\omega - 1)} \tag{5}$$

with $\beta = 2\pi$.

The fact that

$$f_\omega(\tau, \tau') = f_\omega(\tau', \tau + i\beta)$$

or, more explicitly,

$$f_\omega(\tau, \tau') = \mathrm{Tr}[\exp(-\beta\hat{h})\hat{q}(\tau)\hat{q}(\tau')]/\mathrm{Tr}[\exp(-\beta\hat{h})]$$

with

$$\hat{h} = \tfrac{1}{2}\hat{p}^2 + \tfrac{1}{2}\omega^2\hat{q}^2 \qquad \hat{q}(\tau) = \exp(i\tau\hat{h})\hat{q}\exp(-i\tau\hat{h})$$

is, of course, just a quite explicit expression of the Davies-Unruh-Sewell thermality.

The role of the horizon is also evident from equation (4). Only the term $\exp[-i\omega(\tau - \tau')]/2\omega$ appears in the Fulling two-point function

$$F(P, Q) = \langle F, \hat{\varphi}(P)\hat{\varphi}(Q)F \rangle = \int_0^\infty \frac{\exp[-i\omega(\tau - \tau')]}{2\omega}\psi_\omega(\xi)\psi_\omega(\xi')$$

so that, again,

$$\lim_{P\to\partial W_R} F(P, Q) = 0.$$

Only the second term in equation (4) contributes to the coupling between W_R and ∂W_R.

Relevant to our analysis is the observation, easily checked by explicit computation, that the second term in equation (4) can be written as the covariance of a Gaussian stationary process

$$\frac{\cos\omega(\tau-\tau')}{\omega(\exp\beta\omega-1)} = E(q_\omega^\beta(\tau)q_\omega^\beta(\tau'))$$

where

$$q_\omega^\beta(\tau) = (\sqrt{2E}(\omega)/\omega)\sin(\omega\tau-\theta(\omega))$$

with $\theta(\omega)$ and $E(\omega)$ independent random variables with, respectively, uniform distribution on $(0, 2\pi)$ and exponential distribution $\hat\beta\exp(-\hat\beta E)$ on $\mathbb{R}_+$, with $\hat\beta = (\exp\beta\omega-1)/\omega$.

From the formal point of view, the above observation is just Glauber's P-representation [29] of the thermal density matrix of a harmonic oscillator. Notice, however, that, defining the random field

$$\eta^\beta(\tau,\xi) = \int_0^\infty \mathrm{d}\omega\, q_\omega^\beta(\tau)\psi_\omega(\xi)$$

the equality

$$W(P, Q) = F(P, Q) + E(\eta^\beta(P)\eta^\beta(Q))$$

lends itself to a very intuitive interpretation: as opposed to the total decoupling of W_R as described by the Fulling state F, in the Wightman state Ω (quite consistently with the picture of 'energy and entropy flowing through the horizon') the coupling of W_R with ∂W_R is mediated by a classical solution η^β of the field equation, deterministic in its time evolution and random only in its initial conditions, corresponding to the random independent assignments of energies $E(\omega)$ and phases $\theta(\omega)$ to its normal mode amplitudes q_ω^β.

Consistent with the above interpretation (or, equivalently, with the procedure followed by Guerra and Loffredo [26] in order to associate a stochastic process to a thermal state for the harmonic oscillator, by mixing with the Glauber weights the Nelson processes associated to the coherent states appearing in the P-representation of the density matrix, to the Klein-Gordon field on W_R, in the Wightman vacuum state Ω) there appears naturally associated the independent sum

$$\varphi(P) = \varphi_{\mathscr{R}}(P) + \eta^\beta(P)$$

or, equivalently,

$$\varphi(P) = \int_0^\infty \mathrm{d}\omega\left(q_\omega^{\mathscr{R}}(\tau) + \frac{\sqrt{2E(\omega)}}{\omega}\sin(\omega\tau-\theta(\omega))\right)\psi_\omega(\xi). \tag{6}$$

It is, in particular,

$$E(\varphi(P)\varphi(Q)) = \int_0^\infty \mathrm{d}\omega\left(\frac{\exp(-\omega|\tau-\tau'|)}{2\omega} + \frac{\cos\omega(\tau-\tau')}{\omega(\exp\beta\omega-1)}\right)\psi_\omega(\xi)\psi_\omega(\xi').$$

As a final remark, we observe that an ansatz of this form (with of course the appropriate choice of the normal modes) makes sense for static submanifolds of more general manifolds, the numerical value of the parameter β being determined, as in [12], by the requirement that for P on the intersection of past and future horizons the leading contribution as $Q \to P$ coincides with that for the flat case.

Appendix

Equations (4) and (5) are most easily proven starting from the Schwinger points. As noted in [24], Crum's formula [22] gives the following representation of the two-point Schwinger function:

$$\begin{aligned} S(P, Q) &= (2\pi)^{-2} \int \frac{\mathrm{d}^2 K}{K^2 + m^2} \exp[\mathrm{i}K(x-y)] \\ &= (2\pi)^{-1} K_0(m|x-y|) \\ &= \int_0^\infty \mathrm{d}\omega \frac{\exp(-\sigma\omega) + \exp[-(2\pi - \sigma)\omega]}{2\omega[1 - \exp(-2\pi\omega)]} \psi_\omega(z)\psi_\omega(z') \end{aligned}$$

where (θ, τ) and (θ', τ') are the polar coordinates of the points $P = (x^1, x^2)$ and $Q = (y^1, y^2)$ in the Euclidean plane and $\sigma = |\theta - \theta'| \bmod 2\pi$.

Analytic continuation to real time ($x^2 - y^2 \to \mathrm{i}(x^0 - y^0) + 0^+$, corresponding to $\theta - \theta' \to \mathrm{i}(\tau - \tau') + 0^+$) gives the desired result.

References

[1] Nelson E 1985 *Quantum Fluctuations* (Princeton, NJ: Princeton University Press)
[2] Misner C W, Thorpe K S and Wheeler J A 1975 *Gravitation* (San Francisco: Freeman)
[3] Kruskal M D 1960 *Phys. Rev.* **119** 1743
[4] Hawking S W 1975 *Commun. Math. Phys.* **43** 199
[5] Smolin L 1982 *Preprint. On the nature of quantum fluctuations and their relation to gravitation and the principle of inertia* Institute for Advanced Studies, Princeton
[6] Smolin L 1986 *Quantum Concepts in Space and Time* ed R Penrose and C J Sciama (Oxford: Clarendon) p 147
[7] Guerra F and Ruggiero P 1973 *Phys. Rev. Lett.* **31** 1022
[8] Rindler W 1966 *Am. J. Phys.* **34** 1174
[9] Davies P C W 1975 *J. Phys. A: Math. Gen.* **8** 609
[10] Unruh W G 1976 *Phys. Rev.* D **14** 870
[11] Sewell G L 1982 *Ann. Phys., NY* **141** 201
[12] Haag R, Narnhofer H and Stein U 1984 *Commun. Math. Phys.* **94** 219
[13] Fredenhagen K and Haag R 1986 *Preprint. Generally covariant quantum field theory and scaling limit* DESY
[14] Carlen E A 1986 *Preprint. The stochastic mechanics of free scalar fields* Princeton
[15] Reeh H and Schlieder S 1961 *Nuovo Cimento* **22** 1051
[16] Takesaki M 1970 *Tomita's Theory of Modular Hilbert Algebras* (Berlin: Springer)
[17] Bisognano J J and Wichman E H 1975 *J. Math. Phys.* **16** 985
[18] Nelson E 1967 *Dynamical Theories of Brownian Motion* (Princeton, NJ: Princeton University Press)
[19] de Falco D 1984 *Lecture Notes in Mathematics* vol 1055 (Berlin: Springer) p 103
[20] Guerra F 1981 *Phys. Rep.* **77** 263
[21] Fulling S 1973 *Phys. Rev.* D **7** 2850
[22] Erdélyi A 1953 *Higher Transcendental Functions* (New York: McGraw-Hill)
[23] Simon B 1975 *Functional Integration and Quantum Physics* (New York: Academic)

[24] De Angelis G F, de Falco D and Di Genova G 1986 *Workshop on Fundamental Aspects of Quantum Theory, Como 1985* ed A Frigenio and V Gerini (New York: Plenum)
[25] Fényes I 1952 *Z. Phys.* **132** 81
[26] Guerra F and Loffredo M I 1981 *Lett. Nuovo Cimento* **30** 81
[27] Jaekel M T and Pignon D 1984 *J. Phys. A: Math. Gen.* **17** 131
[28] Vilela Mendes R 1986 *Phys. Lett.* **116A** 216
[29] Glauber R J 1963 *Phys. Rev.* **131** 2766